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DIFFERENCE NETWORKS v1.0 EN

Formal Differentics Mathematical Corpus

Project: Formal Differentics

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Abstract

This document introduces Difference Networks, a branch of Formal Differentics that studies systems of interconnected differences represented as mathematical networks. Building upon Difference Information Theory, Difference Algebra, Difference Metrics, Difference Geometry, Difference Topology, and Difference Dynamics, Difference Networks formalizes how differences become linked into large relational structures capable of transmitting information, generating organization, and producing emergent behavior.

The central thesis is that every network can be interpreted as a structured arrangement of differences connected through relations. Nodes represent stabilized difference states, while edges represent active or potential difference relations. Network evolution emerges from the continuous creation, transformation, propagation, and dissolution of differences.

Difference Networks provides a universal framework applicable to physical systems, biological systems, neural networks, social structures, information systems, computational architectures, and abstract mathematical spaces.

1. Introduction

Traditional network theory studies objects and connections.

Differentials proposes an alternative perspective:

Networks are not fundamentally composed of objects.

Networks are fundamentally composed of differences.

Objects emerge as stabilized configurations within networks of differences.

This inversion leads to a Difference-Centered Network Theory.

Instead of:

Object $\rightarrow$ Relation

Differentials adopts:

Difference $\rightarrow$ Relation $\rightarrow$ Network $\rightarrow$ Structure

The network itself becomes a manifestation of organized difference.

2. Fundamental Definitions

Definition 2.1

Difference Node

A Difference Node is a stabilized identifiable state within a Difference Space.

Denoted:

N_i

where:

$$N_i \in DS$$

Definition 2.2

Difference Edge

A Difference Edge is a relation connecting two difference nodes.

Denoted:

$$E_{ij}$$

such that:

$$E_{ij} = D(N_i, N_j)$$

Definition 2.3

Difference Network

A Difference Network is an ordered pair:

$$DN = (N, E)$$

where:

$$N = \{N_1, N_2, \dots\}$$

$$E = \{E_{12}, E_{23}, \dots\}$$

Definition 2.4

Difference Path

A Difference Path is a sequence of connected difference relations.

$$P = (N_1, N_2, \dots, N_k)$$

Definition 2.5

Difference Cycle

A Difference Cycle exists when a difference path returns to its origin.

$$N_1 \rightarrow N_2 \rightarrow \dots \rightarrow N_1$$

3. Axioms of Difference Networks

DN-Axiom 1

No network exists without differences.

DN-Axiom 2

Every edge represents an identifiable difference relation.

DN-Axiom 3

Network complexity increases with the number of distinct relations.

DN-Axiom 4

Difference propagation is constrained by network topology.

DN-Axiom 5

Emergent structures arise from recursive difference interactions.

4. Difference Graph Formalism

A Difference Network can be represented as a graph:

$$G=(V,E)$$

where:

V = Difference Nodes

E = Difference Edges

The Difference Function assigns values:

$$\Delta : E \rightarrow \mathbb{R}$$

such that:

$$\Delta(E_{ij})$$

measures the magnitude of the difference relation.

5. Difference Adjacency Matrix

For a network with n nodes:

$$A=(a_{ij})$$

where:

$$a_{ij} = \Delta(N_i, N_j)$$

if a relation exists,

otherwise:

$$a_{ij} = 0$$

The matrix provides a complete algebraic representation of the network.

6. Difference Connectivity

Definition

Connectivity measures the ability of differences to influence one another.

For node N_i :

$$C(N_i) = \deg(N_i)$$

where:

$$\deg(N_i)$$

is the number of incident difference relations.

Global Connectivity

For network DN:

$$C(DN) = |E|/|N|$$

Higher values indicate denser difference interaction.

7. Difference Centrality

Centrality measures the importance of a difference node.

Degree Centrality

$$CD(N_i) = \deg(N_i)$$

Closeness Centrality

$$CC(N_i) = 1 / \sum d(N_i, N_j)$$

Betweenness Centrality

$$CB(N_i)$$

measures how frequently a node participates in shortest difference paths.

8. Difference Propagation

Differences can spread through networks.

Let:

$$\Delta_t(N_i)$$

denote node state at time t .

Propagation:

$$\Delta_{t+1}(N_i)$$

$$F(\Delta_t(N_1), \dots, \Delta_t(N_n))$$

where F is a propagation operator.

This formalizes:

- information diffusion;
 - signal transmission;
 - causal propagation;
 - network adaptation.
-

9. Difference Clusters

A Difference Cluster is a subset:

$$K \subset DN$$

whose internal relations exceed external relations.

Clusters represent localized structures of organized difference.

Examples:

- communities;
 - ecosystems;
 - neural modules;
 - social groups;
 - knowledge domains.
-

10. Difference Hierarchies

Networks frequently organize into hierarchical structures.

Level 0:

Individual Differences

↓

Level 1:

Difference Nodes

↓

Level 2:

Difference Clusters

↓

Level 3:

Difference Networks

↓

Level 4:

Meta-Networks

This recursive organization produces scalable complexity.

11. Recursive Difference Networks

A network may itself become a node within a higher-order network.

$DN_1 \rightarrow DN_2 \rightarrow DN_3$

This principle formalizes:

Difference Recursion

and directly extends the Differentics principle:

Difference Recursion $\rightarrow$ Diversity

(RECURSITH)

12. CERELATH Networks

Cause-and-effect relations form directed Difference Networks.

$N_i \rightarrow N_j$

means:

N_i contributes causally to N_j

The resulting structure is called a CERELATH Network.

Properties:

- causal chains;
- feedback loops;
- recursive causality;
- dynamic evolution.

13. ADP-TRIFORMER Interpretation

Within ADP-TRIFORMER:

$A \rightarrow \text{DIFFERENTH} \rightarrow B$

represents a minimal network.

Node A

↓

Difference Relation

↓

Node B

Complex networks emerge through recursive expansion of this primitive triadic structure.

Thus:

ADP-TRIFORMER

constitutes the elementary unit of Difference Networks.

14. Emergence Principle

Theorem (Emergent Organization)

Given sufficient connectivity and recursive interaction, a Difference Network may exhibit properties absent from individual nodes.

Symbolically:

Σ Differences

$\neq$

Emergent Whole

Emergence is therefore a network-level phenomenon.

15. Applications

Difference Networks can model:

Physics

interaction networks

Biology

ecological and genetic networks

Neuroscience

neural connectivity

Artificial Intelligence

artificial neural networks

Information Systems

communication architectures

Sociology

social interaction networks

Economics

market structures

Knowledge Systems

semantic networks

16. Relationship to Previous Documents

Difference Networks extends:

- Difference Information Theory (4.1)
- Difference Algebra (4.2)
- Difference Metrics (4.3)
- Difference Geometry (4.4)
- Difference Topology (4.5)
- Difference Dynamics (4.6)

while preparing the foundation for:

Document 4.8

Difference Fields

where networks become continuous fields of distributed differences.

17. Conclusions

Difference Networks establishes the network-theoretic branch of Formal Differentics.

The theory demonstrates that networks can be interpreted as structured systems of interconnected differences. Nodes, edges, paths, clusters, hierarchies, and causal relations emerge naturally from the organization of differences. The framework unifies graph theory, information propagation, causal systems, and emergence under a common difference-centered mathematical formalism.

Difference Networks therefore provides the bridge between local difference interactions and large-scale organized structures, extending the Differentics principle:

Difference Relations generate Structure.

Difference Interactions generate Dynamics.

Difference Recursion generates Diversity.

Difference Connectivity generates Networks.